teaching children mathematics

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

volume 21 + number 4

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The **index** for each volume appears online with the May issue. A cumulative index appears on the NCTM Web site at **www.nctm.org**. *Teaching Children Mathematics* is indexed in *Academic Index*, *Biography Index*, *Contents Pages in Education*, *Current Index to Journals in Education*, *Education Index*, *Exceptional Child Education Resources*, *Literature Analysis of Microcomputer Publications*, *Media Review Digest*, and *Zentralblatt für Didaktik der Mathematik*.

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Teaching Children Mathematics (ISSN 1073-5836) (IPM 1124463) is published monthly except June and July, with a combined December/January issue, by the National Council of Teachers of Mathematics at 1906 Association Drive, Reston, VA 20191-1502. Periodicals postage is paid at Herndon, Virginia, and at additional mailing offices.

POSTMASTER: Send address changes to *Teaching Children Mathematics*, 1906 Association Drive, Reston, VA 20191-1502. Telephone: (703) 620-9840; orders: (800) 235-7566; fax: (703) 476-2970; e-mail: **nctm@nctm.org**; World Wide Web: **www.nctm.org**.

news & views

Mathematical lies we tell students

CHRISTY DANKO GRAYBEAL, PH.D.

With the best of intentions, many teachers find themselves telling students mathematical fibs. We say these half-truths to avoid giving complicated explanations, but children take our statements as absolute truths, and this can lead to misconceptions. What follows is a list of some of the most common "mathematical lies" with explanations for why each is not an exact truth. The Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010) call on students to *attend to precision* (Standards for Mathematical Practice [SMP] 6), and we teachers must do the same by carefully considering our word choices so as to avoid misleading students.

IN MY OPINION

1. ***More*** **means add.**
When students are first learning how to solve word problems, we are tempted to encourage them to look for such keywords as *more* or *together* as clues to determine which operation to use. But keywords are often misleading. For example, consider this problem:

> Jill walked to Bill's house, and then together they walked 4 blocks to the park. In total, Jill walked 7 blocks. How many more blocks did Jill walk than Bill?

MATH IS NOT ABOUT DECODING CLUES BUT ABOUT REASONING AND MAKING SENSE OF SITUATIONS.

This problem could be represented by the equation 4 + ? = 7, but it could also be represented by 7 – 4 = ? Students who have learned that *more* and *together* mean *add* are likely to add seven and four to determine (incorrectly) that Jill has walked eleven blocks. As Van de Walle, Karp, and Bay-Williams (2013) eloquently point out, the use of keywords misleads students to believe that math is about decoding clues instead of reasoning and making sense of situations. Instead, teach students to focus on the problem structure to determine which operations to use.

2. Subtraction means ***take away***.
When reading 5 – 3 aloud, we might casually say, "five *take away* three," but this leads students to believe that subtraction is always synonymous with taking things away. Rather, we should say, "five *minus* three." Consider the following problem:

> Rita has five apples. Alex has three apples. How many more apples does Rita have than Alex?

Even though no "taking away" happens, to solve this problem, one could subtract three from five. Also consider 1000 – 658. If students are thinking of subtraction as only *take away*, they are likely to go directly to the traditional algorithm to solve this. However, students frequently make mistakes when regrouping with zeros. If students are able to think about subtraction in a variety of ways, they may instead think something like the following:

> How far apart are 1000 and 658? If I had a stack of 1000 blocks and a stack of 658 blocks, their difference would be the same as the difference between 999 blocks and 657 blocks. I can find that difference in my head. It is 342.

Flexibility in thinking about operations is essential. Additional problem structures are described in tables 1 and 2 of the Common Core State Standards for Mathematics (CCSSI 2010, pp. 88–89).

3. One plus one ***makes*** two.
Is equivalent to, *equals*, or *is the same as* are all appropriate ways to verbalize the equal sign. We should be careful

not to say *makes*, as this leads students to believe that an equation must have a computation on the left side and an "answer" on the right side. Students who believe this will be confused by such equations as 3 = 2 + 1 or 4 = 4. SMP 6 (CCSSI 2010) explicitly calls on students to use the equal sign consistently and appropriately. See Mann's (2004) article for more discussion of the difficulties students have with regard to understanding the meaning of the equal sign and ways to help students overcome these difficulties.

4. You cannot take a bigger number from a smaller number.

When children are first learning about subtraction that requires regrouping, we often say, "You cannot take a big number from a smaller number." For example, an oral description of 135 – 87 might begin thus: "What is five minus seven? You cannot do that, so we need to regroup . . . ."

But, in fact, we *can* subtract seven from five. The answer is negative two! When children are first learning about subtraction, they are usually limited to knowledge of whole numbers; however, once the domain is expanded to include negative numbers, subtracting a bigger number from a smaller number is possible. To set young children up for later success, we could say,

> What is five minus seven? It would be hard to subtract seven from five with the numbers we have. Later you will learn about numbers that make this easier, but for now we will regroup . . . .

Editor's note: See More4U at the bottom of this page for information on how to access "More Mathematical Lies We Tell Students." Readers may also be interested in the article "13 Rules That Expire" (*TCM* August 2014, pp. 18–25) and the related blog post at http://ow.ly/z3cHs.

REFERENCES

Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

Mann, Rebecca L. 2004. "Balancing Act: The Truth behind the Equal Sign." *Teaching Children Mathematics* 11 (September): 65–69.

Van de Walle, John, Karen S. Karp, and Jennifer M. Bay-Williams. 2013. *Elementary and Middle School Mathematics: Teaching Developmentally*. 8th ed. Boston: Pearson.

Christy Danko Graybeal, Ph.D., graybeal@hood.edu, is an associate professor of education and mathematics at Hood College in Frederick, Maryland. She is a former middle school math teacher and is especially interested in issues related to middle school mathematics teaching. Edited by **Holly Henderson Pinter, hhpinter@wcu.edu**, an assistant professor in the elementary and middle grades education department of Western Carolina University, North Carolina.

more4U

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➝ readers exchange

Quilting area and perimeter

I was so excited when I picked up my copy of the March issue of *TCM*. A group of fourth graders at our school just completed their own quilt—also focusing on perimeter and area as they made it. They began with an exploration of tangrams and how these combined to create shapes of different perimeters but the same area. After a trip to a local fabric store, they designed, created patterns for, and constructed a fabric quilt. I've included a photo of the front. So exciting to see others looking for ways to use area and perimeter in real life situations!

Simultaneously, two other fourth-grade groups were working on projects in which they built model floor plans of 800 square-foot houses and created maps based on scaling up pattern block islands. Interestingly, although the concepts of area and perimeter were our focus, the concept of scale became our most valuable take away.

Gretchen Vice, Math Specialist
The Advent School, Boston, MA
gvice@adventschool.org
www.adventschool.org

COACHES' CORNER

A closer look at Mathematical Practice 6: Attend to precision

BY ROBYN SILBEY, PD AND CAMPUS CONSULTANT

The sixth of the Common Core State Standards for Mathematical Practice (CCSSI 2010) states that mathematically proficient students communicate precisely to others. The standard describes how students share their thinking through both words and symbols. As you guide teachers to ensure precision in daily practice, you may wish to offer strategies based on the actual words found in the standard:

a. Students "use clear definitions in discussion with others and in their own reasoning" (p. 7). After new terms are introduced and defined, they must become embedded into every pertinent discussion about the topic. For example, when operating on numbers, teachers should regularly refer to the results as *sums, differences, products,* and *quotients* rather than *answers*. As teachers use specific mathematical vocabulary in daily active language, students will become comfortable enough with the terms to use them in conversation.

b. Students are "careful about specifying units of measure" (p. 7). Teachers must offer students hands-on practical measuring experiences. Having students develop and use personal referents, such as the length of one's hand span or one's pace, adds sense-making and deep understanding that contribute to precise communication.

c. Students "calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context" (p. 7). Teachers must require that students elaborate on the *how* and *why* of their calculations. Better still are students' clear explanations of how they know their answers make sense.

d. Students "give carefully formulated explanations to each other" (p. 7). Urge teachers to present their students with ample opportunities for discourse. Classroom conversations sharpen students' skills in vocabulary development and precise communication while solidifying conceptual understanding.

Attending to precision involves being able to articulate one's thinking at every stage of the problem-solving process clearly enough for others to understand and replicate it.

Questions? Comments? Contact **robyn@robynsilbey.com**.